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# Propagation of dispersion–nonlinearity-managed solitons in an inhomogeneous erbium-doped fiber system

**A Mahalingam**[1], **K Porsezian**[2,4], **M S Mani Rajan**[2] **and A Uthayakumar**[3]

[1] Department of Physics, Anna University, Chennai-600 025, India
[2] Department of Physics, Pondicherry University, Puducherry-605 014, India
[3] Department of Physics, Dr. Ambedkar Govt. Arts College, Chennai-600 039, India

E-mail: mahabs22@yahoo.com, ponz.phy@pondiuni.edu.in, rajeshwarimurugan@yahoo.com and uthayk@yahoo.com

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**Abstract**

In this paper, a generalized nonlinear Schrödinger–Maxwell–Bloch model with variable dispersion and nonlinearity management functions, which describes the propagation of optical pulses in an inhomogeneous erbium-doped fiber system under certain restrictive conditions, is under investigation. We derive the Lax pair with a variable spectral parameter and the exact soliton solution is generated from the Bäcklund transformation. It is observed that stable solitons are possible only under a very restrictive condition for the spectral parameter and other inhomogeneous functions. For various forms of the inhomogeneous dispersion, nonlinearity and gain/loss functions, construction of different types of solitary waves like classical solitons, breathers, etc is discussed.

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## 1. Introduction

In recent years, the investigation of the nonlinear dynamics of inhomogeneous systems has attracted special attention because these systems are considered to be more realistic than their corresponding homogeneous counterparts. Optical fiber solitons are considered to be the most important milestone on the path of communication technology. This is because optical solitons are formed as a result of a perfect balance between the group velocity dispersion

[4] Author to whom any correspondence should be addressed.

(GVD) and the nonlinear effect, which are considered to be the major problems in high-speed optical fiber communication. The GVD causes the temporal broadening of the optical pulse due to the frequency dependence of the index of refraction. When an intense optical pulse is propagated through a silica fiber, the medium tends to behave nonlinearly because of the intensity-dependent refractive index $n = n_0(\omega) + n_2|E|^2$, and the phase of the pulse gets modulated which is called self-phase modulation (SPM) [1]. An optical pulse which experiences both these effects is governed by the nonlinear Schrödinger (NLS) equation. However, for an inhomogeneous fiber, a generalized inhomogeneous nonlinear Schrödinger equation (GINLSE) model is considered to be the most important realistic and universal nonlinear model.

The propagation of the optical solitons in an ideal fiber is governed by the nonlinear Schrödinger (NLS)-type equations. To avoid the problems caused by the electronic amplifiers, all-optical communication systems are in vogue now and erbium-doped fiber amplifiers (EDFA) are widely used for this purpose. When erbium (Er) is doped with the core of the optical fibers, then the nonlinear wave propagation can have both the effects due to silica and Er impurities. Er impurities provide the self-induced transparency (SIT) effect to the optical pulse whereas the silica material gives the NLS soliton effect [2, 3]. Erbium is selected because the energy difference between the two levels is nearly equal to that of the frequency at which present day optical signals are transmitted. The coherent interaction effect is due to resonant absorption which can balance the optical losses in the fiber medium. The pulse propagation in erbium-doped two-level resonant atoms is called SIT soliton-type pulse propagation. In 1967, McCall and Hahn [4] described a special type of lossless pulse propagation in two-level resonant media. They showed that if the energy difference between the two levels of the media coincides with the frequency of the optical signal, then coherent absorption takes place and the medium becomes optically transparent to that particular wavelength. In erbium-doped fibers, the resultant solitons are collectively called nonlinear Schrödinger–Maxwell–Bloch (NLS-MB) solitons. This type of soliton pulse propagation was theoretically shown for the first time by Maimistov and Manykin [5] in 1983 and many other results were also reported on these NLS-MB-type fiber systems [6–12]. Nakazawa *et al* [13, 14] experimentally observed the coexistence of NLS solitons and SIT solitons in erbium-doped resonant fibers. In [2, 3, 15], the possibility of coexistence of NLS and SIT solitons with higher order linear and nonlinear effects has been investigated.

In recent years, the problem of nonlinear wave propagation in inhomogeneous media has been found to be of great interest which has a wide range of applications. The inhomogeneity in the fiber mainly arises due to two factors: (i) the variation in the lattice parameters of the fiber medium and (ii) variation of the fiber geometry [16]. Of late, the effect of these inhomogeneities on the propagation of solitary wave pulses in an optical fiber has produced considerable activity among researchers. In particular, for the theoreticians, the question is to analyze the way in which the behavior of solitons is affected and to find out whether these inhomogeneous systems are still integrable like their homogeneous counterparts.

However, there are a number of factors which affect the dynamics of optical solitons and the conditions for the generation of optical solitons in real fibers. For instance, some important factors are the dissipative losses leading to the damping of soliton amplitude without changing its velocity [17], higher order dispersion effects [18], various inhomogeneities of fiber [19], alternating conditions of exploitation of optical lines, etc. Keeping this in mind, in the present work, a most generalized inhomogeneous NLS-MB system is proposed and analyzed for the solitary wave propagation.

## 2. The generalized inhomogeneous nonlinear Schrödinger–Maxwell–Bloch (GINLS-MB) model

The solution of the NLS-type equation in an inhomogeneous medium is of great importance for investigating wave propagation in various types of physical situations such as plasma physics, nonlinear optics, condensed matter, and so on. Serkin *et al* introduced the GINLS equation and obtained the one- and two-soliton solution through the Lax pair technique [20]. In this work, we have modified the GINLS equation to suit with erbium-doped fibers, wherein the effect of SIT should be included and the governing equation is now called the GINLSE-MB equation of the following form:

$$\begin{aligned} &\mathrm{i}\, Q_z + \frac{D(z)}{2} Q_{tt} + R(z)|Q|^2 Q + F_1(z,t)Q + F_2(z)Q - 2\mathrm{i}A(z)\langle p\rangle = 0 \\ &p_t = \mathrm{i}\, p[2\omega - t\theta] + 2Q\eta \qquad \eta_t = -\frac{R(z)}{D(z)}(p^* Q + pQ^*) \end{aligned} \tag{1}$$

where $Q(z,t)$ is the complex envelope of the field; $F_1(z,t)$ is related to time-dependent phase modulation; $F_2(z)$ is the Wronskian of $D(z)$ and $R(z)$; $A(z)$ is the parameter accounting for the interaction between silica and doped atoms; $p(z,t)$ is the measure of the polarization of the resonant medium; $\eta(z,t)$ denotes the extent of the population inversion, which are given by $\nu_1\nu_2^*$ and $|\nu_1|^2 - |\nu_2|^2$ respectively, $\nu_1$ and $\nu_2$ being the wavefunctions of the two energy levels of the resonant atoms; $D(z)$ represents the group velocity dispersion (GVD) function; $R(z)$ is the nonlinearity management function; and $\theta(z)$ is the phase shift parameter [20]. The angular bracket represents averaging over the entire frequency range. Thus

$$\langle p(z,t)\rangle = \int_{-\infty}^{\infty} p(z,t;\omega)h(\omega)\,\mathrm{d}\omega \tag{2}$$

$$\int h(\omega)\,\mathrm{d}\omega = 1 \tag{3}$$

where $h(\omega)$ is the uncertainty in the energy levels.

In equation (1), if we take $D(z) = R(z) = A(z) =$ constant and $\alpha = 0$, then the system reduces to the well-known integrable NLS-MB equation and the details of these investigations have already been discussed in the introduction.

The above system is the most general one to describe the optical pulse propagation in an inhomogeneous nonlinear dispersive doped fiber. In this paper, this system is solved by means of the Lax pair and Bäcklund transformation (BT) technique, as described below.

## 3. Lax pair

In this section, we aim to generate the soliton solutions with the help of the associated Lax pair and Bäcklund transformation. Here, we modify the Lax pair for the GINLS system proposed by Serkin *et al* [20] to be suitable for the GINLS-MB system. By applying the AKNS formalism, we can construct the linear eigenvalue problem for equation (1) as follows:

$$\psi_t = U\psi, \qquad \psi_z = V\psi \tag{4}$$

where $U$ and $V$ are

$$U = \begin{pmatrix} -\mathrm{i}\lambda & q \\ -q^* & \mathrm{i}\lambda \end{pmatrix} \tag{5}$$

$$V = \mathrm{i}\begin{pmatrix} \frac{1}{2}Dq\,q^* - \alpha t & D\left(\frac{1}{2}q_t - \mathrm{i}\,\theta t q\right) \\ D\left(\frac{1}{2}q_t^* + \mathrm{i}\theta t q^*\right) & -\frac{1}{2}Dq\,q^* + \alpha t \end{pmatrix} - \mathrm{i}\lambda D\begin{pmatrix} \theta t & \mathrm{i}q \\ -\mathrm{i}q^* & -\theta t \end{pmatrix}$$
$$- \mathrm{i}\lambda^2 D\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{\mathrm{i}A(z)}{(\lambda+\omega)}\begin{pmatrix} \eta' & -p' \\ -p^{*\prime} & -\eta' \end{pmatrix} \tag{6}$$

where

$$q(z,t) = \sqrt{\frac{R}{D}}Q(z,t)\exp\frac{1}{2}(\mathrm{i}\,t^2\theta), \tag{6a}$$

$$p'(z,t) = \sqrt{\frac{R}{D}}p(z,t)\exp\frac{1}{2}(\mathrm{i}\,t^2\theta), \tag{6b}$$

and

$$\eta'(z,t) = \eta(z,t).$$

Equation (1) can be obtained from the compatibility condition $U_z - V_t + [U,V] = 0$, if we assume that the inhomogeneous parameters satisfy the following conditions:

$$\begin{aligned} F_1(z,t) &= -\left[2t\alpha + \frac{t^2}{2}\frac{\mathrm{d}\theta}{\mathrm{d}z} + \frac{D}{2}(\mathrm{i}\theta - t^2\theta^2)\right] \\ F_2(z) &= \frac{\mathrm{i}}{2}\left[\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}z} - \frac{1}{D}\frac{\mathrm{d}D}{\mathrm{d}z}\right] \\ \theta(z) &= \frac{-W[R(z),D(z)]}{D(z)^2R(z)} \\ W[R(z),D(z)] &= RD_z - DR_z. \end{aligned} \tag{6c}$$

It should be mentioned here that the constraints imposed on the inhomogeneous parameter by equation (6*c*) are difficult to be materialized in a real optical fiber. However, these constraints are necessary to obtain a soliton solution through the Lax pair and a comprehensive analysis of inhomogeneous fibers with realistic parameters could be carried out only by numerical methods which is outside the scope of this work. The compatibility condition is satisfied if and only if $\lambda$ satisfies the following relation:

$$\lambda = \mathrm{e}^{\int_0^z D(z')\theta(z')\,\mathrm{d}z'}\left(\lambda(0) + \int_0^z \mathrm{e}^{-\int_0^z D(z')\theta(z')\,\mathrm{d}z'}\alpha(z)\,\mathrm{d}z'\right). \tag{7}$$

From the Lax pair, soliton solutions can be generated by using the auto-Bäcklund transformation method as shown below.

## 4. Bäcklund transformation

Bäcklund transformation (BT) allows one to generate multisoliton solutions of nonlinear evolution equations [6]. We have derived the Bäcklund transformation from the time evolution equations of the eigenfunctions. To obtain the BT of equation (1), let us write down equation (4) in the Riccati form. By introducing new variables (pseudo potentials),

$$\Gamma = \frac{\psi_1}{\psi_2} \tag{8}$$

so that the spatial and temporal linear eigenvalue problems reduce to the Riccati equations:

$$\begin{aligned} \Gamma_t &= q - 2\mathrm{i}\lambda\Gamma + q^*\Gamma^2 \\ \Gamma_z &= -2\mathrm{i}\alpha t\Gamma - 2\mathrm{i}\lambda D\theta t\Gamma - 2\mathrm{i}\lambda^2 D\Gamma + \frac{2\mathrm{i}A(z)\Gamma}{(\lambda+\omega)}. \end{aligned} \tag{9}$$

Similarly, defining $\Gamma' = \frac{\psi_1'}{\psi_2'}$ will satisfy the equations similar to equation (9). On choosing $\Gamma' = \frac{1}{\Gamma^*}$, and after some mathematical manipulation, we obtain

$$q' - q = \frac{-4\gamma\Gamma}{(1 + |\Gamma|^2)} \tag{10}$$

where the primed quantities correspond to $N$-soliton solutions and unprimed quantities correspond to $(N-1)$ soliton solutions, and $\lambda(0) = \mu + \mathrm{i}\gamma$. To construct the soliton solution of equation (1), we start with the zero soliton solution $q(0) = 0$, $p = 0$ and $\eta = \pm 1$ (pure states). By substituting the above conditions in the spatial and temporal eigenvalue problems, the explicit form of one-soliton solution is obtained. This procedure can obviously be continued and it furnishes in a recursive manner all the higher order soliton solutions, and the associated wavefunction can also be generated. By using Bäcklund transformation, the single soliton solution of the GINLS-MB equation is obtained as follows:

$$Q(z, t) = \sqrt{\frac{D(z)}{R(z)}}\frac{2\gamma}{\sqrt{\sigma}} \operatorname{sech}(f_1) \exp(-\mathrm{i}\xi_1) \tag{11}$$

where $f_1$ and $\xi_1$ are

$$f_1 = 2\gamma t + \int_0^z \left(4\mu\gamma D + 2tD\theta\gamma + \frac{2A\gamma}{\gamma^2 + (\mu - \omega)^2}\right) \mathrm{d}z + \delta_1$$

$$\xi_1 = 2\mu t + \frac{t^2\theta}{2} + \int_0^z \left(2\mu^2 D - 2\gamma^2 D + 2t\alpha + 2D\theta t\mu - \frac{2A(\mu - \omega)}{\gamma^2 + (\mu - \omega)^2}\right) \mathrm{d}z + \delta_2$$

where $\delta_1$ and $\delta_2$ are independent of both $z$ and $t$. In equation (11), if we take $D(z) = R(z) = A(z) =$ constant and $\alpha = 0$, then the solution becomes the NLS-MB soliton solution reported earlier. $\mu$ and $\gamma$ are the velocity and amplitude parameters of the soliton pulse, respectively. Once the one-soliton solution is known, it is possible to generate multisoliton solutions in a systematic way, but for inhomogeneous systems, this approach is too complicated. At this juncture, it should be mentioned that by using suitable transformation, many of the integrable inhomogeneous equations have been transformed to their corresponding homogeneous counterparts. However, because of the complexity of our problem, we are not able to find such a transformation for our case.

Having obtained the soliton solution of equation (1), our next aim is to analyze the impact of dispersion and nonlinear management by considering various forms for both GVD and nonlinearity parameters as the functions of $z$. Such an approach may find fruitful applications in what can be called dispersion–nonlinearity-managed (DNM) soliton systems.

## 5. Results and discussions

To study the pulse propagation characteristics, we can design the optimal soliton propagation systems by appropriately choosing the different forms of distributed parameters according to the specific problem. With the entry of dispersion management, the GVD coefficient is no longer a constant, but a function of the propagation distance ($z$). It is a well-known fact that when the group velocity dispersion is varied even slightly, the behavior of the pulse changes drastically from its regular one. The underlying principle of soliton dispersion management is the robustness of optical solitons. The important idea in the DM soliton systems is to minimize the path average dispersion while maintaining local dispersion. This can be achieved by periodically inserting fiber Bragg gratings with opposite dispersion along the transmission line. Pulse compression is the other interesting area where the effect of inhomogeneities effectively reduces the pulse width. The strong dispersion management technique offers the possibility of controlling fiber nonlinearity and suppressing specific nonlinear effects such as the self-phase modulation and inter-channel crosstalk in wavelength-division-multiplexed fiber systems [21].

Here, we consider some important systems that are currently being discussed in the literature and also some new systems for which we get some exotic solitons like boomerang solitons, phase-shifting solitons, reported for the first time for bright solitons.

### *5.1. Periodically distributed amplification system*

Of late, researchers are focusing on periodic distributed systems because of their potential applications in long-distance dispersion-managed soliton communication systems. Equation (1) includes mainly two arbitrary distributed functions $D(z)$ and $R(z)$. Thus, by choosing different forms for them, one can analyze various solitary wave propagations. First, we consider a periodic distributed amplification system with a varying group velocity dispersion parameter $D(z)$ and a nonlinearity parameter $R(z)$ as follows [22–25]:

$$D(z) = \frac{1}{d_0}\exp(kz)R(z) \qquad R(z) = R_0 + R_1\sin(gz) \tag{12}$$

where $R_0$, $R_1$ and $g$ are the parameters which define Kerr nonlinearity and $d_0$ is the parameter related to the initial peak power in the system, respectively. Here, for the sake of simplicity, we take $R_0 = 0$, $d_0 = 1$ and $g = 1$. When $k = 0$, it corresponds to the case of fibers without any loss or gain. When $k < 0$, it represents a dispersion-decreasing fiber, and similarly, $k > 0$ corresponds to dispersion-increasing fibers. In this paper, we consider the above three scenarios and analyze the corresponding solitary wave propagation in detail.

*Case* (i) $k = 0$. For this case, the Wronskian becomes zero for the above choice of parameters, in which case, there is no gain or loss. Hence, the pulse does not suffer any broadening or compression but an overall phase change due to the SIT effect induced by the doped atoms and an oscillating behavior, known as snaking due to high intensity or velocity as shown in figures 1(*a*)–(*f*).

For the sake of simplicity, the function $A(z)$ is assumed to be in the same form as that of $D(z)$. The propagation of solitary waves can be controlled by varying the numerical values of the parameters involved. Initially, the parameter $\mu$, which denotes the velocity of the solitons, is varied from low to high, but the intensity is kept constant. At lower values of $\mu$, one could get classical solitons, but when it is increased, the solitons undergo slight oscillations as shown in figure 1(*b*). When the velocity is increased further, periodic oscillations in time, the resulting behavior is known as snaking, which occurs as depicted in figure 1(*c*). A similar behavior is observed when the parameter $\gamma$, which is responsible for the intensity of the pulse, is varied, keeping $\mu$ as a constant. If the intensity is changed from low values to high values, the soliton changes its profile from its classic nature to snaking nature, which is clearly shown in figures 1(*d*) and (*e*). It is also evident that the amplitude of the soliton does not change and that the periodic oscillations occur at constant intervals, mainly because of the absence of gain or loss.

The above plots were plotted by considering the form of $A(z)$ to be the same as that of the dispersion function. Next, we tried to analyze the solitary waves for various forms for $A(z)$. Our graphical simulations suggest that when $A(z)$ is taken as a constant, there is an overall phase shift for the pulse, as shown in figure 1(*f*), but otherwise the soliton does not suffer from any phase change at all in the absence of the SIT effect ($A(z) = 0$). Hence it can be concluded

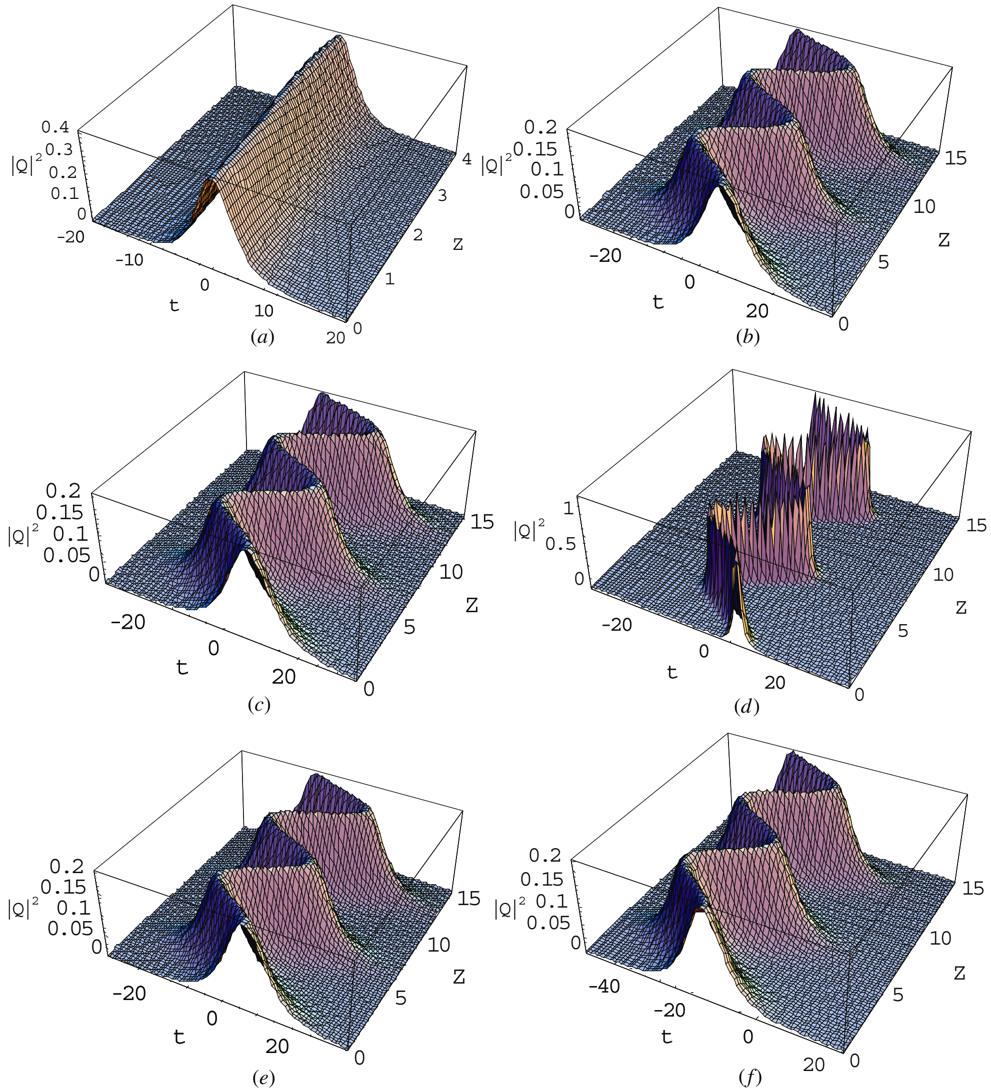



**Figure 1.** The evolution of the bright one-soliton solution (*a*) when the velocity is low ($\mu = 0.005$) and (*b*) when the velocity is increased ($\mu = 3.5$) with $\gamma = 0.2$. The evolution of the bright one-soliton solution (*c*) when the intensity is low ($\gamma = 0.1$) and (*d*) when the intensity is increased ($\gamma = 0.6$) with $\mu = 3.5$. (*e*) The evolution of the bright one-soliton solution when the velocity is high ($\mu = 3.5$) and $\gamma = 0.1$ with $A(z) = \frac{1}{d_0}\exp(kz)R(z)$, and (*f*) parameters same as plot (*e*), but with $A = 0.5$.

that an overall phase shift for the soliton occurs only when $A(z)$ is taken as a constant or zero in which case the effect of SIT is absent. For other forms of $A(z)$, it is observed that the profile of the soliton is changed.

*Case* (ii) $k < 0$. This case stands for a dispersion-decreasing fiber. Similar to case (i), we analyzed the propagation characteristics of the soliton by numerically controlling the different physical parameters. Without any loss of generality, we have assigned the values of some parameters as $R_0 = 0$, $d_0 = 1$ and $g = 1$. From the plots in figures 2(*a*) and (*b*), it is observed

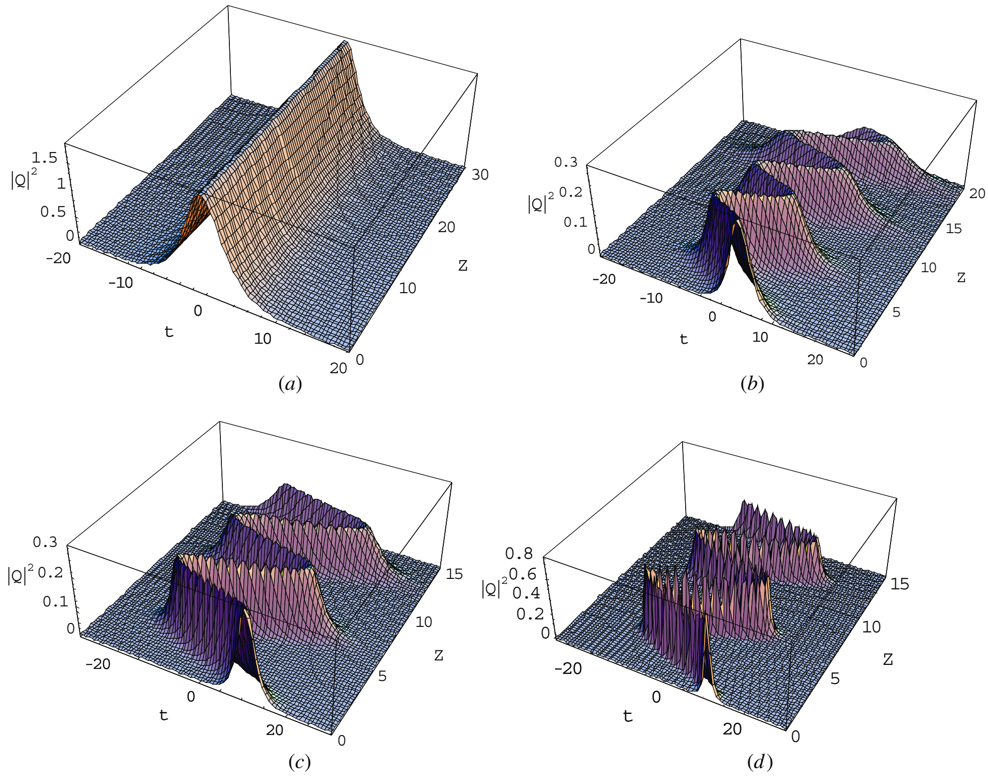



**Figure 2.** The evolution of the bright one-soliton solution (*a*) when the velocity is low ($\mu = 0.005$) and (*b*) when the velocity is high ($\mu = 0.6$) with $\gamma = 0.15$. (*c*) The evolution of the bright one-soliton solution when the intensity is high ($\gamma = 0.15$) and (*d*) ($\gamma = 0.4$) with $\mu = 7.6$.

that the width of the pulse decreases as a result of negative chirping due to the negative value of $k$. The periodic oscillations are more pronounced when the velocity or intensity of the pulse is high as could be seen from figures 2(*b*) and (*c*), respectively. Here also an overall phase shift is produced for constant $A(z)$ as shown in figure 2(*d*).

*Case* (iii) $k > 0$. This condition is applied for dispersion-increasing fibers. Because of the positive $k$ value, the pulse gets broadened while propagating in the fiber medium. This could be clearly seen from figures 3(*a*)–(*d*). The width of the pulse increases as the dispersion parameter is now having a positive value when the distance increases. At lower velocities as well as intensities, the periodic oscillations are absent and periodic oscillations are started for higher values of $\gamma$ as illustrated in figures 3(*b*) and (*c*). In this case also, an overall phase shift is observed for constant $A(z)$ as shown in figure 3(*d*). Note that the system parameter values are the same as in case (ii).

### 5.2. *Pulse compression*

Let us consider the pulse compression of an optical pulse in a dispersion-decreasing optical fiber. For this purpose, we assume that the GVD and the nonlinearity functions are distributed

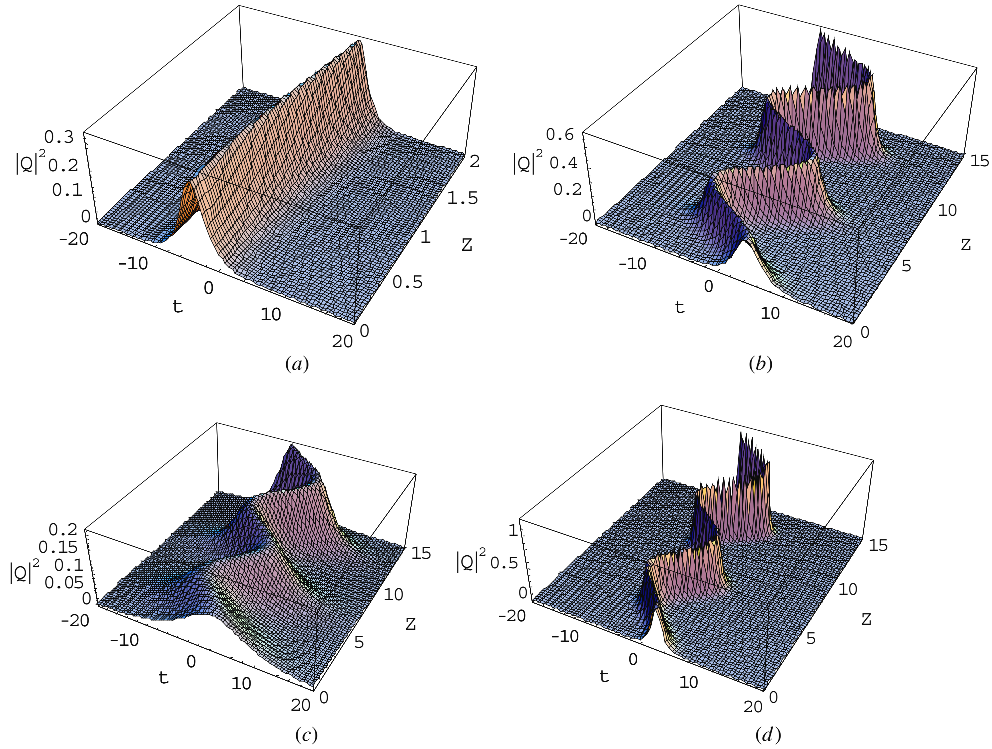



**Figure 3.** The evolution of the bright one-soliton solution (*a*) when the velocity is low ($\mu = 0.0001$) and (*b*) when the velocity is high ($\mu = 8.2$) with $\gamma = 0.15$. (*c*) The evolution of the bright one-soliton solution when the intensity is high ($\gamma = 0.005$) and (*d*) $\gamma = 0.3$ with $\mu = 6.2$.

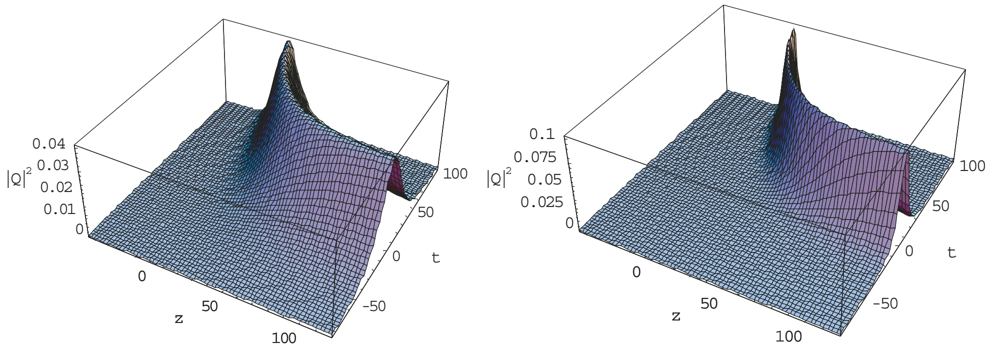
in the form given as follows [21, 26–28]:

$$D(z) = d\exp(-gz) \qquad R(z) = r\exp(-kz) \tag{13}$$

where $d$ and $g$ are related to GVD. $r$ and $k$ are the parameters which describe the nonlinearity. For $g < 0$, solitons are compressed exponentially during the propagation. For $g > 0$ solitons get broadened. For $k = -g$, width of the pulse remains unchanged. When $g > 0$ and $k < 0$, the system describes the dispersion-decreasing fiber. If we choose $g < 0$ and $k > 0$, the system describes the dispersion increasing fiber. In figure 4(*b*), we can see that the soliton gets compressed during its propagation due to $k \neq 0$; the velocity and time shift of the soliton vary with the dispersion distribution during the soliton pulse propagation. This property implies that we can control the interaction between two solitons by suitably controlling each soliton velocity.

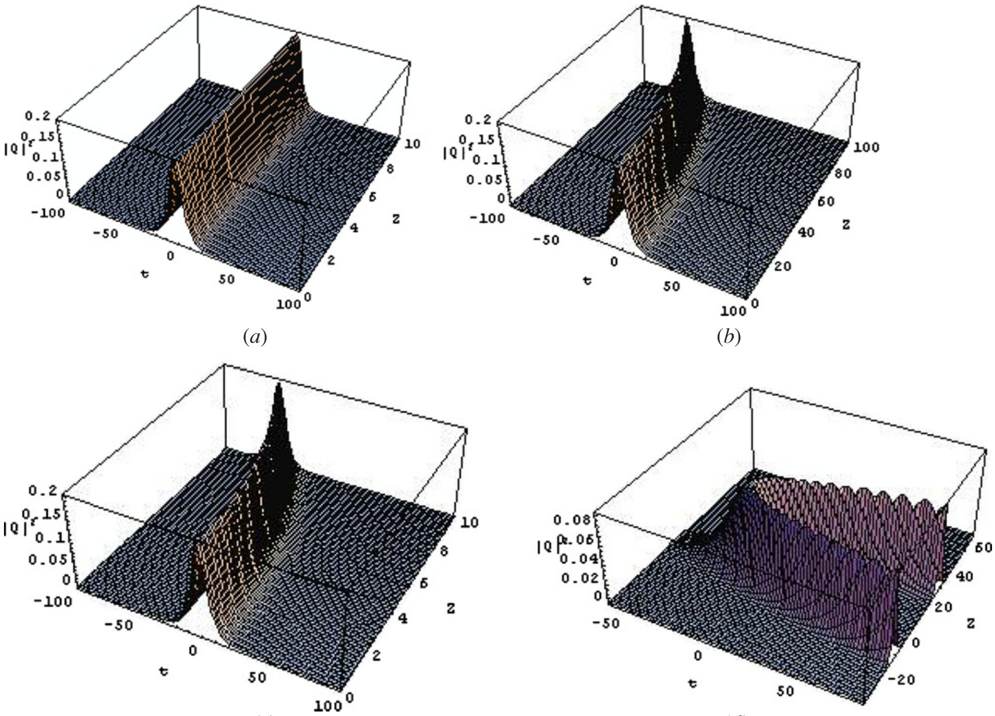
### *5.3. Phase-shifting solitons*

Next, we analyze the case when both dispersion and nonlinearity parameters vary linearly with the distance $z$. Depending on the intensity and velocity parameters, we obtained different kinds of solitons as exemplified in the figures of this section. Initially, the parameter $A(z)$, which represents the effect of doping, was kept constant. When the velocity parameter $\mu$ is small, one can obtain classical soliton propagation which propagates without any shape or




(*a*) (*b*)

**Figure 4.** The evolution of the bright one-soliton solution (*a*) when $A(z) = d\exp(-gz)$ and (*b*) when $A(z) = 0$ with $d = r = 1$, $g = 0.03$ and $k = 0.025$.




(*a*) (*b*)

(*c*) (*d*)

**Figure 5.** The evolution of classic one-soliton solution when $A(z) = 1$; $R(z) = D(z) = z$; $\alpha = 0.35$; $\gamma = 0.1$; m $= 0.005$. (*a*) For small distances and (*b*) for longer distances. (*c*) Same as (*a*) except m $= 0.5$. (*d*) Boomerang soliton with $\alpha = 0.35$; $\gamma = 0.045$; $\mu = 1$.

phase change up to a particular distance, but for the same parameters, the phase of the soliton changes after traveling a long distance, as shown in figures 5(*a*) and (*b*), respectively. When the distance is further increased, the soliton bends more and an increase in $\mu$ results in more phase shift (figure 5(*c*)), while an increase in $\gamma$ value, which is responsible for intensity, results in modulational instability. When the parameter $A(z)$ is taken to be equal to $z$, it is observed that more phase shift occurs.

In a recent paper [29], Li and Chen have obtained a particular type of dark solitons, which they named as boomerang solitons. In the anomalous dispersion regime, one would get bright solitons and we have found that such bright boomerang solitons can exist in inhomogeneous optical systems as depicted in figure 5(*d*) when the GVD and SPM parameters are taken in the form $D(z) = R(z) = A(z) = a - bz$ with $a = 1$ and $b = 0.15$. Interestingly, these boomerang solitons change their direction when $D(z) = R(z) = A(z) = a + bz$.

## 6. Conclusion

The optical solitons provoke a great interest theoretically and experimentally for their potential applications like the optical fiber transmission system, ultrafast optical switches, pulse compression, etc. With the consideration of varying dispersion and nonlinearity parameters, we have considered a GINLS-MB equation which describes the propagation of optical pulses in an inhomogeneous erbium-doped fiber. To obtain solitary wave solutions, the linear eigenvalue problem has been constructed using the AKNS procedure and the Lax pair has been obtained. The one-soliton solution has been obtained using Bäcklund transformation. The propagation of the GINLS-MB soliton is analyzed for some specific cases for the inhomogeneous and nonlinearity parameters.

With the consideration of varying dispersion and nonlinearity, equation (1) describes the propagation of optical pulse in an inhomogeneous fiber. In practical case, the model is of primary interest not only for the compression and amplification of optical solitons in an inhomogeneous system, but also for the stable transmission of soliton pulses with effective control of inhomogeneous parameters. It should be noted here that stable solitons were observed for this system only under very restrictive conditions for the inhomogeneous parameters involved, which would make it difficult in real fibers. It is our belief that in future the difficulties in realizing a fiber with inhomogeneous parameters as described in this work could be overcome in the manufacturing process itself. Analyses of inhomogeneous fiber systems are still sporadic and hence this work is to be considered as a small but important step in this direction.

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